Eur. Phys. J. B **13**, 271–277 (2000)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Balance-equation approach to terahertz-field-driven magnetotransport in semiconductors

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Received 17 May 1999

Abstract. We investigate the magnetotransport in semiconductors under the influence of a dc or slowly-varying electric field, an intense polarized radiation field of terahertz frequency, and a uniform magnetic field, being in arbitrary directions and having arbitrary strengths. Effective force- and energy-balance equations are derived by using a gauge that the magnetic field and the high-frequency radiation field are described by a vector potential and the dc or slowly-varying field by a scalar potential, and by distinguishing the slowly-varying velocity from the rapidly-oscillating velocity related to the high-frequency field. These equations, which include the elastic photon process and all orders of multiphoton absorption and emission processes, are applied to the examination of the effect of a terahertz radiation on the magnetophonon resonance of the longitudinal resistivity in the transverse configuration in nonpolar and polar semiconductors. We find that the previous zero-photon resonance peaks are suppressed by the irradiation of the terahertz field, while many new peaks, which may be related to multiphoton absorption and emission processes, emerge and can become quite distinct, at moderately strong radiation field.

PACS. 72.20.My Galvanomagnetic and other magnetotransport effects – 72.30.+q High-frequency effects; plasma effects – 72.20.Ht High-field and nonlinear effects

1 Introduction

The magnetic field has played a crucial role in the development of our understanding of condensed matters. In particular, deep insight into the properties of semiconductors has been generated by phenomena related to magnetotransport such as Hall effect, Shubnikov-de Haas-van Alphen oscillation, cyclotron resonance, and magnetophonon resonance. Since first proposed theoretically by Gurevich and Firsov [1], magnetophonon resonance (MPR) has been extensively investigated, both experimentally and theoretically in semiconductors [2]. It arises from the unique feature of the electron density of states as a result of the Landau quantization and the resonance occurs when the energy separation between two Landau levels matches the optic phonon energy, leading to oscillating behavior in a variety of transport and optical properties of the system as functions of the magnetic field. MPR can exist in both nondegenerate and degenerate semiconductors at relatively high temperatures when optic-phonon scattering contributes and has become one of the main instruments of semiconducting compound spectroscopy.

The recent development of the free-electron laser, which provides a continuously tunable source of linearly polarized terahertz (THz) electromagnetic radiation of high intensity, has made it possible to irradiate a semiconductor with an intense far-infrared field under various conditions. Nonlinear dynamics of the electron gas driven by THz radiation fields in semiconductors has become a central focus of many experimental and theoretical studies in the literature [3–12]. Interesting and unusual phenomena related to multiphoton processes have been observed in resonant tunneling systems and in miniband superlattices [3–5]. When a magnetically-quantized semiconductor is exposed to an intense electromagnetic radiation of THz frequency, the magnetophonon resonance behavior is expected to be affected significantly by the radiation field due to multiphoton emission and absorption. These interesting phenomena can only be disclosed within a fully nonlinear theory capable of dealing with the situation having a strong dc electric field, an intense high-frequency electromagnetic field and a strong dc magnetic field coexisting.

Theoretically, in the presence of a THz radiation field and a magnetic field, most of previous investigations on transport were carried out based on linear response theory [13]. To our knowledge, the recent work of Bryksin and Kleinert [14] is the only exception, who studied the influence of a strong THz irradiation on transport in

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semiconductor superlattices in the presence of dc electric and magnetic fields in the Wannier-Stark and Landau quantization regime in longitudinal-parallel configuration (dc electric and magnetic fields and the high-frequency radiation field are all perpendicular to the layer of the superlattice).

The purpose of the present paper is to develop a balance-equation approach to terahertz-driven magnetotransport in semiconductors with a dc or slowly-varying electric field, an intense polarized radiation field of THz frequency and a uniform magnetic field, being in arbitrary directions and having arbitrary strengths, applied simultaneously in the system. Using a gauge that the magnetic field and the high-frequency radiation field are described by a vector potential and the dc or slowly-varying field by a scalar potential, we are able to distinguish the slowly-varying velocity from the rapidly-oscillating velocity related to the high-frequency field. Following the procedure in deriving the Lei-Ting balance equations [15], and considering the fact that relevant transport quantities are measured as averages over a time interval much longer than the period of the radiation field, we obtain a set of momentum and energy balance equations, which formally are the extension of the original balance equation in the presence of the magnetic field [16] to include all the multiphoton processes.

These equations are applied to the examination of magnetophonon resonance in the longitudinal resistivity in hot-electron transport in a semiconductor driven by a THz radiation having various frequency and strength and subjected to a dc bias. We will focus on the transverse configurations in nonpolar and polar semiconductors at moderately high lattice temperatures. Numerical calculation shows that the previous magnetophonon resonance peaks in the longitudinal resistivity as a function of the strength of the magnetic field in the absence of high-frequency field, are suppressed by the irradiation of the terahertz electromagnetic field, while many new peaks, which correspond to multiple photon emission and absorption processes, emerge and may become quite distinct, under the influence of a moderately strong radiation field.

2 The electron system subject to a dc magnetic field and a high-frequency field

We consider an isotropic three-dimensional (3D) electron system, consisting of N electrons in a unit volume, having constant effective mass m and charge e. These electrons are interacting with each other and also coupled with phonons and scattered by randomly distributed impurities in the lattice.

To model the situation of a magnetically-quantized system irradiated by an intense terahertz (THz) field, we assume that a constant magnetic field $\mathbf{B}$, a dc (or slowly varying) electric field $\mathbf{E}_0$, and a uniform sinusoidal radiation field of frequency ω and amplitude $\mathbf{E}_\omega$, $\mathbf{E}_\omega \sin(\omega t)$, are applied simultaneously in the system. $\mathbf{B}$, $\mathbf{E}_0$ and $\mathbf{E}_\omega$ can be in arbitrary directions and of arbitrary strengths. We describe these electric and magnetic fields ($\mathbf{B}$, $\mathbf{E}_0$ and $\mathbf{E}_\omega \sin(\omega t)$) by means of a vector potential $\mathbf{A}(\mathbf{r},t)$ and a scalar potential $\varphi(\mathbf{r})$ of the form

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}(\mathbf{r}) + (\mathbf{E}_\omega/\omega)\cos(\omega t), \tag{1}$$

$$\varphi(\mathbf{r}) = -\mathbf{r}\cdot\mathbf{E}_0, \tag{2}$$

in which $\mathbf{A}(\mathbf{r})$ is the vector potential of the uniform magnetic field $\mathbf{B}$:

$$\nabla \times \mathbf{A}(\mathbf{r}) = \mathbf{B}. \tag{3}$$

In the presence of these electric and magnetic fields the Hamiltonian of the system reads

$$H = H_{\rm eE} + H_{\rm ei} + H_{\rm ep} + H_{\rm ph}. \tag{4}$$

Here

$$H_{\rm eE} = \sum_j \left[\frac{1}{2m}\left(\mathbf{p}_j - e\mathbf{A}(\mathbf{r}_j,t)\right)^2 + \varphi(\mathbf{r}_j)\right] + H_{\rm ee} \tag{5}$$

is the Hamiltonian of the electrons under the influence of the electric and magnetic fields with $H_{\rm ee}$ being the electron-electron Coulomb interaction, $H_{\rm ei}$ and $H_{\rm ep}$ are, respectively, the electron-impurity and electron-phonon couplings, and $H_{\rm ph}$ stands for the phonon Hamiltonian. In equation (5), $\mathbf{r}_j$ and $\mathbf{p}_j$ are the coordinate and momentum of the jth electron, and the spin-splitting is not included for simplicity.

Introducing the center-of-mass (CM) momentum and coordinate variables $\mathbf{P}$ and $\mathbf{R}$, and the relative electron momentum and coordinate variables $\mathbf{p}'_j$ and $\mathbf{r}'_j$ ($j = 1, ...N$)

$$\mathbf{P} = \sum_j \mathbf{p}_j, \qquad \mathbf{R} = \frac{1}{N}\sum_j \mathbf{r}_j, \tag{6}$$

$$\mathbf{p}'_j = \mathbf{p}_j - \frac{1}{N}\mathbf{P}, \quad \mathbf{r}'_j = \mathbf{r}_j - \mathbf{R}, \tag{7}$$

we can separate the Hamiltonian $H_{\rm eE}$ into a CM part $H_{\rm CM}$ and a relative electron part $H_{\rm er}$:

$$H_{\rm eE} = H_{\rm CM} + H_{\rm er},$$

$$H_{\rm CM} = \frac{1}{2Nm}\left(\mathbf{P} - Ne\mathbf{A}(\mathbf{R},t)\right)^2 - Ne\mathbf{E}_0\cdot\mathbf{R} \tag{8}$$

$$H_{\rm er} = \sum_j \frac{1}{2m}\left(\mathbf{p}'_j - e\mathbf{A}(\mathbf{r}'_j)\right)^2 + \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{e^2}{|\mathbf{r}'_i - \mathbf{r}'_j|}. \tag{9}$$

Note that both the dc electric field $\mathbf{E}_0$ and the high-frequency electric field $\mathbf{E}_\omega \sin(\omega t)$ exert only on the center-of-mass, and relative electrons do not directly see the existence of either one of them ($\mathbf{E}_0$ or $\mathbf{E}_\omega$). Furthermore, in the presence of the radiation field, $H_{\rm ei}$ and $H_{\rm ep}$ have exactly the same expressions as those given in reference [15], in terms of CM coordinate $\mathbf{R}$ and the density operator of the relative electrons,

$$\rho_{\mathbf{q}} = \sum_j {\rm e}^{{\rm i}\mathbf{q}\cdot\mathbf{r}'_j}. \tag{10}$$

3 Force- and energy-balance equations

Based on the Heisenberg equation of motion, we can obtained the velocity (operator) of the center-of-mass, $\mathbf{V}$, which is the rate of change of the CM coordinate $\mathbf{R}$, and the acceleration operator of the center-of-mass, $\dot{\mathbf{V}}$, which is the rate of change of CM velocity:

$$\mathbf{V} = -\mathrm{i}\,[\mathbf{R}, H] = \frac{1}{Nm}\,(\mathbf{P} - Ne\mathbf{A}(\mathbf{R}, t)) = \frac{1}{Nm}\,(\mathbf{P} - Ne\mathbf{A}(\mathbf{R})) - \mathbf{v}_\omega \cos(\omega t), \tag{11}$$

with $\mathbf{v}_\omega \equiv e\mathbf{E}_\omega/(m\omega)$, and

$$\dot{\mathbf{V}} = -\mathrm{i}\,[\mathbf{V}, H] + \frac{\partial \mathbf{V}}{\partial t} = \frac{e}{m}\mathbf{E}_0 + \frac{e}{m}(\mathbf{V}\times\mathbf{B}) + \frac{\mathbf{F}}{Nm} + \omega \mathbf{v}_\omega \sin(\omega t), \tag{12}$$

with

$$\mathbf{F} = -\mathrm{i}\sum_{\mathbf{q},a} u(\mathbf{q})\,\mathbf{q}\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot(\mathbf{R}-\mathbf{r}_a)}\rho_{\mathbf{q}} - \mathrm{i}\sum_{\mathbf{q},\lambda} M\,(\mathbf{q},\lambda)\,\mathbf{q}\,\phi_{\mathbf{q}\lambda}\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{R}}\rho_{\mathbf{q}}. \tag{13}$$

We can also obtain the rate of change of the relative electron energy:

$$\dot{H}_{\mathrm{er}} = -\mathrm{i}\,[H_{\mathrm{er}}, H] = -\mathrm{i}\sum_{\mathbf{q},a} u(\mathbf{q})\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot(\mathbf{R}-\mathbf{r}_a)}\dot{\rho}_{\mathbf{q}} - \mathrm{i}\sum_{\mathbf{q},\lambda} M(\mathbf{q},\lambda)\,\phi_{\mathbf{q}\lambda}\,\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{R}}\dot{\rho}_{\mathbf{q}}. \tag{14}$$

In equations (13, 14) $\mathbf{r}_a$ and $u(\mathbf{q})$ are the impurity position and its potential, $M(\mathbf{q},\lambda)$ is the matrix element due to coupling between electrons and a phonon of wavevector $\mathbf{q}$ in branch λ having energy $\Omega_{\mathbf{q}\lambda}$, $\phi_{\mathbf{q}\lambda} \equiv b_{\mathbf{q}\lambda} + b^\dagger_{-\mathbf{q}\lambda}$ stands for the phonon field operator, and $\dot{\rho}_{\mathbf{q}} \equiv -\mathrm{i}[\rho_{\mathbf{q}}, H_{\mathrm{er}}]$.

Following reference [15], we treat the CM coordinate $\mathbf{R}$ and velocity $\mathbf{V}$ classically, and, by neglecting their small fluctuations we will regard them as time-dependent expectation (or average) values of the CM coordinate and velocity, $\mathbf{R}(t)$ and $\mathbf{V}(t)$. Furthermore, the statistical average of the CM velocity consists of a slowly-varying part

$$\left\langle \frac{1}{Nm}\,(\mathbf{P} - Ne\mathbf{A}(\mathbf{R}))\right\rangle_0 = \mathbf{v}_0 \tag{15}$$

and a rapid oscillating part $-\mathbf{v}_\omega\cos(\omega t)$. We write

$$\mathbf{V}(t) = \mathbf{v}_0 - \mathbf{v}_\omega\cos(\omega t), \tag{16}$$

and thus

$$\mathbf{R}(t) = \int_{t_0}^{t}\mathbf{V}(s)\mathrm{d}s + \mathbf{R}(t_0). \tag{17}$$

Therefore, $H_{It} \equiv H_{\mathrm{ei}} + H_{\mathrm{ep}}$, $\dot{\mathbf{V}}$ and $\dot{H}_{\mathrm{er}}$ are time-dependent operators in the relative-electron–phonon systems.

These results indicate that, as far as the relative electron system is concerned, everything is the same as that discussed in reference [12] except that the vector potential related to the uniform magnetic field, $\mathbf{A}(\mathbf{r})$, shows up in H_{er}. Therefore we can proceed in the same way as in reference [12] to obtain the density matrix of the relative electron system. For semiconductors with relatively high carrier density, it is adequate to solve the Liouville equation for the density matrix of the relative electron-phonon system by starting from an initial state at time $t = -\infty$, in which the phonon system is in equilibrium at the lattice temperature T and the relative electron system is in equilibrium at an electron temperature T_{e}:

$$\hat{\rho}|_{t=-\infty} = \hat{\rho}_0 = \frac{1}{Z}\mathrm{e}^{-H_{\mathrm{er}}/T_{\mathrm{e}}}\mathrm{e}^{-H_{\mathrm{ph}}/T}. \tag{18}$$

With the density matrix thus obtained to the first order in $H_{\mathrm{ei}} + H_{\mathrm{ep}}$, we can derive the momentum- and energy-balance equation by taking the statistical average of the operator equations (12) and (14). If all the transport quantities are measured as averages over a time interval much longer than the period of the terahertz field, any part oscillating at frequency ω and its harmonics will be washed out. Therefore, after the statistical average and this "long-time interval" average, only slowly varying part of a transport quantities survive. For instance, the average of the rate of change of CM velocity, $\dot{\mathbf{V}}$, can be identified as the time derivative of the slowly-varying CM velocity, $\mathrm{d}\mathbf{v}_0/\mathrm{d}t$, and the average of the rate of change of the relative electron energy, $\dot{H}_{\mathrm{er}}$, can be identified as the time derivative of the slowly-varying relative electron energy, $\mathrm{d}\mathcal{E}_{\mathrm{e}}/\mathrm{d}t$. Thus, we are left with the following force- and energy-balance equations containing only the slowly-varying quantities:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v}_0 = \frac{e\mathbf{E}_0}{m} + \frac{e}{m}\mathbf{v}_0\times\mathbf{B} + \frac{\mathbf{F}_{\mathrm{i}}}{Nm} + \frac{\mathbf{F}_{\mathrm{p}}}{Nm}, \tag{19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}_{\mathrm{e}} = -\mathbf{v}_0\cdot(\mathbf{F}_{\mathrm{i}} + \mathbf{F}_{\mathrm{p}}) - W + S_{\mathrm{p}}. \tag{20}$$

Here

$$\mathbf{F}_{\mathrm{i}} = n_{\mathrm{i}}\sum_{\mathbf{q}}|u(\mathbf{q})|^2\,\mathbf{q}\sum_{n=-\infty}^{\infty}J_n^2(\mathbf{q}\cdot\mathbf{r}_\omega)\,\Pi_2(\mathbf{q},\omega_0 - n\omega), \tag{21}$$

and

$$\mathbf{F}_{\mathrm{p}} = 2\sum_{\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2\,\mathbf{q} \times \sum_{n=-\infty}^{\infty}J_n^2(\mathbf{q}\cdot\mathbf{r}_\omega)\,\Lambda(\mathbf{q},\lambda,\Omega_{\mathbf{q}\lambda} + \omega_0 - n\omega) \tag{22}$$

are frictional forces experienced by the CM due to impurity and phonon scatterings in the presence of magnetic field and terahertz radiation field,

$$W = 2\sum_{\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2\,\Omega_{\mathbf{q}\lambda} \times \sum_{n=-\infty}^{\infty}J_n^2(\mathbf{q}\cdot\mathbf{r}_\omega)\,\Lambda(\mathbf{q},\lambda,\Omega_{\mathbf{q}\lambda} + \omega_0 - n\omega) \tag{23}$$

is the energy-transfer rate from the electron system to the phonon system, and

$$S_{\rm p} = n_{\rm i}\sum_{\mathbf q}|u(\mathbf q)|^2 \times \sum_{n=-\infty}^{\infty} J_n^2(\mathbf q\cdot\mathbf r_\omega)\, n\omega\, \Pi_2(\mathbf q,\omega_0-n\omega) + 2\sum_{\mathbf q,\lambda}|M(\mathbf q,\lambda)|^2 \times \sum_{n=-\infty}^{\infty} J_n^2(\mathbf q\cdot\mathbf r_\omega)\, n\omega\, \Lambda(\mathbf q,\lambda,\Omega_{\mathbf q\lambda}+\omega_0-n\omega) \quad (24)$$

is the net rate of the energy the electron system gains from the radiation field through impurity- and phonon-assisted multiphoton (absorption and emission) processes ($n = \pm1,\pm2,...$). In equations (21–24), $\omega_0 \equiv \mathbf q\cdot\mathbf v_0$, $\mathbf r_\omega \equiv \mathbf v_\omega/\omega = e\mathbf E_\omega/(m\omega^2)$, $J_n(x)$ is the Bessel function of order n, $\Pi_2(\mathbf q,\Omega)$ is the imaginary part of the Fourier representation of the electron density-density correlation function $\Pi(\mathbf q,t)$ in the presence of the magnetic field, which is defined by

$$\Pi(\mathbf q,t) = -{\rm i}\theta(t)\langle[\rho_{\mathbf q}(t),\rho_{-\mathbf q}(0)]\rangle_0, \quad (25)$$

in which $\langle..\rangle_0$ stands for the statistical average with reference to the initial density matrix (18). And we have denoted

$$\Lambda(\mathbf q,\lambda,\Omega) \equiv \Pi_2(\mathbf q,\Omega)\left[n\left(\frac{\Omega_{\mathbf q\lambda}}{T}\right) - n\left(\frac{\Omega}{T_{\rm e}}\right)\right], \quad (26)$$

with $n(x) \equiv 1/[\exp(x)-1]$ being the Bose function.

Except an additional classical Lorentz-force term showing up in the force-balance equation, the force- and the energy-balance equations and the expressions for $\mathbf F_{\rm i}$, $\mathbf F_{\rm p}$, W and $S_{\rm p}$, are formally the same as those for hot-electron transport driven by a terahertz radiation in the absence of a magnetic field [12]. The major effect of the magnetic field on terahertz-field-driven transport is included in the density-density correlation function $\Pi(\mathbf q,\Omega)$ of the electrons, which is drastically modified when the magnetic field is strong. Under random-phase-approximation (RPA), the effect of the Coulomb interaction between electrons gives rise to a dynamical screening, such that [16] $\Pi(\mathbf q,\Omega) = \Pi_0(\mathbf q,\Omega)/\epsilon(\mathbf q,\Omega)$, where $\epsilon(\mathbf q,\Omega)$ is the RPA dielectric function, and $\Pi_0(\mathbf q,\Omega)$ is the density correlation function of noninteracting electrons in the presence of the magnetic field.

Without loss of generality, we assume that the magnetic field is in the z direction: $\mathbf B = (0,0,B)$. The energy spectrum of electron forms the Landau levels ($n = 1,2,...$)

$$\varepsilon_n(k_z) = (n+\frac{1}{2})\omega_{\rm c} + \frac{k_z^2}{2m}, \quad (27)$$

where $\omega_{\rm c} = |eB|/m$ is the cyclotron frequency. The density correlation function of noninteracting electrons can be written in the Landau representation:

$$\Pi_0(\mathbf q,\Omega) = \frac{1}{2\pi l^2}\sum_{n,n'} C_{n,n'}(l^2q_\parallel^2/2)\,\Pi_0(n,n',q_z,\Omega). \quad (28)$$

Here $l^2 = 1/|eB|$ is the magnetic length, $q_\parallel^2 \equiv q_x^2+q_y^2$, and

$$C_{n,n'}(x) \equiv \frac{n_2!}{n_1!}x^{n_1-n_2}{\rm e}^{-x}[L_{n_2}^{n_1-n_2}(x)]^2, \quad (29)$$

with $n_1 = \max(n,n')$, $n_2 = \min(n,n')$, and $L_m^l(x)$ being the associated Laguerre polynomial. Without including the Landau level broadening, $\Pi_0(n,n',q_z,\Omega)$ function was given in reference [16]. Its imaginary part $\Pi_{02}(n,n',q_z,\Omega)$, to which the dominant contribution comes from the small $|q_z|$ region, exhibits a delta-function-type peak $\Pi_{02}(n,n',0,\Omega) \sim \delta(\Omega+(n'-n)\omega_{\rm c})$ at $q_z = 0$. People generally introduce a Landau level broadening in order to eliminate the logarithm divergence in the linear magnetoresistivity in the transverse configuration. The selection of the broadening parameter, however, is somewhat arbitrary. We note that when system is biased by a finite dc current ($\mathbf v_0 \neq 0$), the nonlinearity itself results in an equivalent broadening and the divergence is eliminated naturally [17]. In this paper, we will focus on hot-electron magnetotransport having a sufficient dc current bias to avoid the need for an additional broadening of the Landau levels.

If $\mathbf E_0$ is a constant (dc) field, we can seek for the steady-state solution to equations (19, 20) with the steady-state values $\mathbf v_0$ and $T_{\rm e}$ determined by the following equations:

$$Ne\mathbf E_0 + Ne\mathbf v_0\times\mathbf B + \mathbf F = 0, \quad (30)$$

$$\mathbf v_0\cdot\mathbf F + W - S_{\rm p} = 0, \quad (31)$$

in which $\mathbf F = \mathbf F_{\rm i} + \mathbf F_{\rm p}$. These equations are valid for $\mathbf E_0$ (or $\mathbf v_0$) and $\mathbf E_\omega$ in arbitrary directions. It is easily seen that for the following 4 configurations the frictional force $\mathbf F$ will be in the direction opposite to $\mathbf v_0$ and we can write $\mathbf v_0\cdot\mathbf F = v_0F(v_0)$, $W = W(v_0)$ and $S_{\rm p} = S_{\rm p}(v_0)$ with $F(v_0)$, $W(v_0)$ and $S_{\rm p}(v_0)$ being functions of $v_0 = |\mathbf v_0|$ only: (a) longitudinal-parallel configuration $\mathrm L_\parallel$: $\mathbf v_0\|\mathbf B$, $\mathbf E_\omega\|\mathbf B$; (b) longitudinal-perpendicular configuration $\mathrm L_\perp$: $\mathbf v_0\|\mathbf B$, $\mathbf E_\omega\perp\mathbf B$; (c) transverse-parallel configuration $\mathrm T_\parallel$: $\mathbf v_0\perp\mathbf B$, $\mathbf E_\omega\|\mathbf B$; (d) transverse-perpendicular configuration $\mathrm T_\perp$: $\mathbf v_0\perp\mathbf B$, $\mathbf E_\omega\|\mathbf v_0$. For these configurations, it is convenient to define a resistivity function

$$\rho(v_0) = -\frac{F(v_0)}{N^2e^2v_0}, \quad (32)$$

and an energy-dissipation resistivity function

$$\rho_E(v_0) = \frac{W(v_0)-S_{\rm p}(v_0)}{N^2e^2v_0^2}\,. \quad (33)$$

The energy-balance equation (31) is written as

$$\rho(v_0) - \rho_E(v_0) = 0, \quad (34)$$

which determines the electron temperature for given v_0. In the longitudinal configurations ($L_\parallel$ and $L_\perp$), the resistivity is given by

$$\rho_{zz} \equiv \frac{E_0}{Nev_0} = \rho(v_0). \tag{35}$$

In the transverse configurations ($T_\parallel$ and $T_\perp$), the longitudinal resistivity is given by

$$\rho_{xx} \equiv \frac{\mathbf{E}_0 \cdot \mathbf{v}_0}{Nev_0^2} = \rho(v_0). \tag{36}$$

In view of the behavior of $\Pi_{02}(\mathbf{q}, \Omega)$ function, the dominant contribution to the integration in expressions (21–24) for $\mathbf{F}_\mathrm{i}$, $\mathbf{F}_\mathrm{p}$, W and S_p, generally comes from the small $|q_z|$ region (in comparison with dominant $q_\parallel$), where the argument $\mathbf{q}\cdot\mathbf{r}_\omega$ of the Bessel functions is generally small if $\mathbf{E}_\omega \| \mathbf{B}$, such that $J_n^2(\mathbf{q}\cdot\mathbf{r}_\omega)$ ($n \neq 0$) is small and $J_0^2(\mathbf{q}\cdot\mathbf{r}_\omega) \approx 1$. This indicates that in the parallel configurations ($T_\parallel$ and $L_\parallel$) the effect of the radiation field on magnetotransport is weaker than in the case of $\mathbf{E}_\omega \perp \mathbf{B}$. On the other hand, since it is known that, in the absence of a high-frequency field, the magnetophonon resonance in the longitudinal configuration appears much smaller than in the transverse configuration, we expect a similar conclusion for photon-assisted magneto-phonon resonance. In the following, we will focus on $T_\perp$ configuration.

4 Nonpolar semiconductors

We first consider nonpolar semiconductors at relatively high lattice temperature with dominant optical-phonon deformation potential scattering: $|M(\mathbf{q},\lambda)|^2 = D^2/(2d_\mathrm{c}\Omega_\mathrm{o})$, where Ω_o is the optic phonon frequency, d_c is the mass density of the lattice and D is the shift of the band edge per unit relative displacement of the two sublattices in associate with the optic phonon mode. Assuming nondegenerate distribution and ignoring the screening, we can write the imaginary part of the electron density correlation function in the magnetic field, $\Pi_{02}(\mathbf{q},\Omega)$, in a closed-form integral representation in terms of elementary functions [18]. In the case of zero spin-splitting,

$$\Pi_{02}(\mathbf{q},\Omega) = -\left(\frac{m}{2\pi}\right)^{3/2} \omega_\mathrm{c} T_\mathrm{e}^{1/2} \mathrm{e}^{\mu/T_\mathrm{e}} \frac{\sinh(\Omega/2T_\mathrm{e})}{\sinh(\omega_\mathrm{c}/2T_\mathrm{e})} \times \int_0^\infty \mathrm{d}y \cos(\Omega y/2) \exp\left[-(1+T_\mathrm{e}^2y^2)\frac{q_z^2}{8mT_\mathrm{e}}\right] \times \exp\left[-\frac{q_\parallel^2}{2m\omega_\mathrm{c}} \frac{\cosh(\omega_\mathrm{c}/2T_\mathrm{e}) - \cos(\omega_\mathrm{c} y/2)}{\sinh(\omega_\mathrm{c}/2T_\mathrm{e})}\right], \tag{37}$$

where μ is the chemical potential, which is related to the carrier density N as

$$\mathrm{e}^{\mu/T_\mathrm{e}} = \frac{(2\pi)^{3/2} N \sinh(\omega_\mathrm{c}/2T_\mathrm{e})}{m^{3/2} T_\mathrm{e}^{1/2} \omega_\mathrm{c}} \,. \tag{38}$$

Using this expression and carrying out the q_y and q_z integrations, we have

$$\rho(v_0) = \frac{D^2 m \omega_\mathrm{c}^{1/2} T_\mathrm{e}^{1/2}}{4\pi^2 e^2 d_\mathrm{c} \Omega_\mathrm{o} \sinh(\Omega_\mathrm{o}/2T) v_0} \times \sum_{n=-\infty}^{\infty} \int q_x \mathrm{d}q_x J_n^2(q_x r_\omega)\, A_\mathrm{D}(q_x, n\omega), \tag{39}$$

$$\rho_E(v_0) = -\frac{D^2 m \omega_\mathrm{c}^{1/2} T_\mathrm{e}^{1/2}}{4\pi^2 e^2 d_\mathrm{c} \Omega_\mathrm{o} \sinh(\Omega_\mathrm{o}/2T) v_0^2} \times \sum_{n=-\infty}^{\infty} (\Omega_\mathrm{o} - n\omega) \int \mathrm{d}q_x J_n^2(q_x r_\omega)\, A_\mathrm{D}(q_x, n\omega), \tag{40}$$

in which

$$A_\mathrm{D}(q_x, n\omega) \equiv \int_0^\infty \mathrm{d}y \frac{\cos[(\Omega_\mathrm{o} + q_x v_0 - n\omega)y/2]}{\sqrt{1+T_\mathrm{e}^2 y^2}} \times \sinh\left(\frac{\Omega_\mathrm{o} + q_x v_0 - n\omega}{2T_\mathrm{e}} - \frac{\Omega_\mathrm{o}}{2T}\right) \times \sqrt{\frac{\sinh(\omega_\mathrm{c}/2T_\mathrm{e})}{\cosh(\omega_\mathrm{c}/2T_\mathrm{e}) - \cos(\omega_\mathrm{c} y/2)}} \times \exp\left[-\frac{q_x^2}{2m\omega_\mathrm{c}} \frac{\cosh(\omega_\mathrm{c}/2T_\mathrm{e}) - \cos(\omega_\mathrm{c} y/2)}{\sinh(\omega_\mathrm{c}/2T_\mathrm{e})}\right]. \tag{41}$$

We use $v_\mathrm{o} \equiv (\Omega_\mathrm{o}/m)^{1/2}$ and $E_\mathrm{o}^* \equiv (m\Omega_\mathrm{o}^3)^{1/2}/|e|$, as the velocity scale and the field-strength scale. For a typical system with $m = 0.082 m_\mathrm{e}$ (m_e is the free electron mass) and $\Omega_\mathrm{o} = 30.5\,\mathrm{meV}$, we have $v_\mathrm{o} = 2.5 \times 10^7\,\mathrm{cm/s}$ and $E_\mathrm{o}^* = 55.7\,\mathrm{kV/cm}$. Under the condition that the semiconductor is dc-current biased to $v_0 = 0.03 v_\mathrm{o}$ along the x-direction, and the lattice temperature is $T = 0.4\Omega_\mathrm{o}$, we calculate the electron temperature T_e and the longitudinal resistivity $\rho_{xx} = \rho(v_0)$ from equations (34, 36) in the absence and in the presence of a high-frequency sinusoidal electric field. The ac fields polarize along the x-direction, having angular frequency $\omega = 0.1\Omega_\mathrm{o}$ and 3 different strengths: $E_\omega = 0.005E_\mathrm{o}^*, 0.0075E_\mathrm{o}^*$, and $0.01E_\mathrm{o}^*$. The calculated value of the longitudinal resistivity ρ_{xx}, normalized by its value in the absence of the magnetic field and high-frequency field, ρ_0, is plotted in Figure 1 as a function of $\omega_\mathrm{c}/\Omega_\mathrm{o}$.

In the absence of high-frequency field ($E_\omega = 0$), we have the conventional hot-electron magnetophonon resonance [19]: the longitudinal resistivity ρ_{xx} (due to nonpolar optic phonon scattering) as a function of the magnetic field exhibits a marked hump consisting of two peaks around each center position determined by the condition $M\omega_\mathrm{c} = \Omega_\mathrm{o}$ with a small dip at the center, and a deep valley between two neighboring center positions. The two-peak appearance of the hump around an expected resonant position ($M = 1$ or 2) is due to the existence of

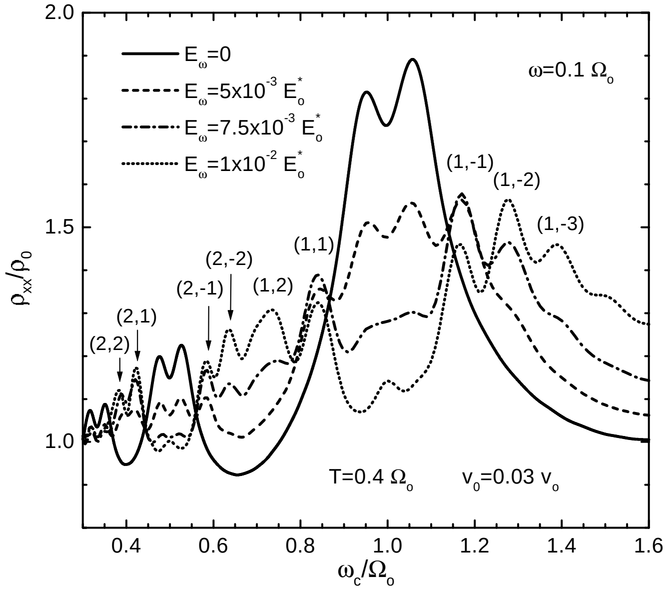

Fig. 1. The multiphoton-magnetophonon resonance in the longitudinal resistivity ρ_{xx} of a nonpolar semiconductor driven by a dc bias velocity $v_0 = 0.03v_o$ and by a radiation field of frequency $\omega = 0.1\Omega_o$ having three different amplitudes $E_\omega = 0.005, 0.0075$, and $0.01E_o^*$ in the $T_\perp$ configuration. The lattice temperature is $T = 0.4\Omega_o$. The optic phonon deformation potential scattering is assumed to dominate. Here ρ_0 represents the resistivity of the system at the same lattice temperature with the same dc bias velocity in the absence of the magnetic field ($B = 0$) and in the absence of the radiation field ($E_\omega = 0$). The multiphoton peaks are indicated by two integers (M, n) with reference to the resonant condition $M\omega_c + n\omega = \Omega_o$.

the finite dc current bias, which shifts the resonance condition from $M\omega_c = \Omega_o$ to $M\omega_c = \Omega_o \pm \bar{q}_x v_0$ ($\bar{q}_x$ stands for an effective average wavevector), and thus is relevant to the value of bias velocity v_0. We refer these humps as zero-photon humps. When an ac field of frequency $\omega = 0.1\Omega_o$ is applied, we see that (1) the humps descend and valleys ascend, and (2) additional resonant peaks emerge on both sides of each zero-photon hump. The positions of these side peaks can be somewhat related to the condition $M\omega_c + n\omega = \Omega_o$, $(n = 1, 2, 3, ..)$. These peaks are referred to as multiphoton resonant peaks and identified by two integers (M, n). With increasing the strength of the high-frequency field, the zero-photon humps further decrease and the multiphoton peaks, (1,1), (1,-1), (1,2), (1,-2), (1,-3), (2,1), (2,-1), and (2,-2) show up clearly. In the case of $E_\omega = 0.01E_o^*$, the previous humps become valleys, and the above-mentioned multiphoton peaks appear quite distinct.

On the other hand, the electron temperature as a function of the magnetic field, though oscillates and shows multiphoton peaks, the overall effect is quite small at this strength of the radiation field as shown in Figure 2.

5 Polar semiconductors

In the case of polar semiconductors, the dominant scattering mechanism is the Fröhlich scattering by polar optic phonons: $|M(\mathbf{q}, \lambda)|^2 = e^2\Omega_{\rm LO}(\kappa_\infty^{-1} - \kappa_0^{-1})/(2\epsilon_0 q^2)$, where $\Omega_{\rm LO}$ is the frequency of the polar optic phonon, and κ_0 and κ_∞ are respectively static and high-frequency dielectric constant of the material.

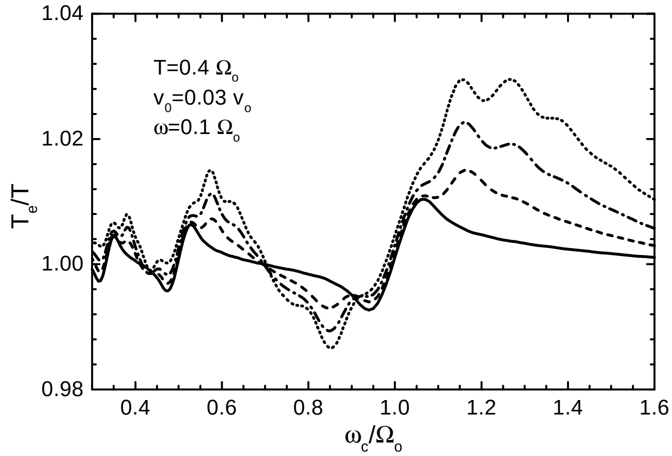

Fig. 2. The electron temperature T_e/T *versus* ω_c/Ω_o, for the same system and under the same condition as described in Figure 1.

Making use of the expression for $\Pi_{02}(n, n', q_z, \Omega)$ given in reference [16], in the nondegenerate limit we can write $\rho(v_0)$ and $\rho_E(v_0)$ for the $T_\perp$ configuration as

$$\rho(v_0) = \rho_{\rm LO}^* \frac{{\rm e}^{\mu/T_{\rm e}}}{mv_0}\frac{\omega_{\rm c}}{\Omega_{\rm LO}} \times \sum_{n=-\infty}^{\infty}\sum_{n_1,n_2}\int_0^\infty {\rm d}q_y \int_0^\infty {\rm d}q_z \int {\rm d}q_x \frac{q_x}{(q_\parallel^2 + q_z^2)q_z} \times J_n^2(q_x r_\omega) C_{n_1 n_2}(l^2 q_\parallel^2/2) A_{\rm LO}^{n_1 n_2}(q_x, q_z, n\omega), \quad (42)$$

$$\rho_E(v_0) = -\rho_{\rm LO}^* \frac{{\rm e}^{\mu/T_{\rm e}}}{mv_0^2}\frac{\omega_{\rm c}}{\Omega_{\rm LO}} \times \sum_{n=-\infty}^{\infty}\sum_{n_1,n_2}\int_0^\infty {\rm d}q_y \int_0^\infty {\rm d}q_z \int {\rm d}q_x \frac{\Omega_{\rm LO} - n\omega}{(q_\parallel^2 + q_z^2)q_z} \times J_n^2(q_x r_\omega) C_{n_1 n_2}(l^2 q_\parallel^2/2) A_{\rm LO}^{n_1 n_2}(q_x, q_z, n\omega), \quad (43)$$

with

$$A_{\rm LO}^{n_1 n_2}(q_x, q_z, n\omega) = \exp\left(-\frac{(n_1 + 1/2)\omega_{\rm c}}{T_{\rm e}} - \frac{[(n_2 - n_1)\omega_{\rm c} + \Omega_{\rm LO} + q_x v_0 - n\omega + q_z^2/2m]^2}{2T_{\rm e} q_z^2/m}\right) \times \frac{\exp[(\Omega_{\rm LO} + q_x v_0 - n\omega)/T_{\rm e}] - \exp(\Omega_{\rm LO}/T)}{\exp(\Omega_{\rm LO}/T) - 1}, \quad (44)$$

and

$$\rho_{\rm LO}^* \equiv \frac{m^3\Omega_{\rm LO}^2}{4\pi^4 N^2 \epsilon_0}\left(\frac{1}{\kappa_\infty} - \frac{1}{\kappa_0}\right). \quad (45)$$

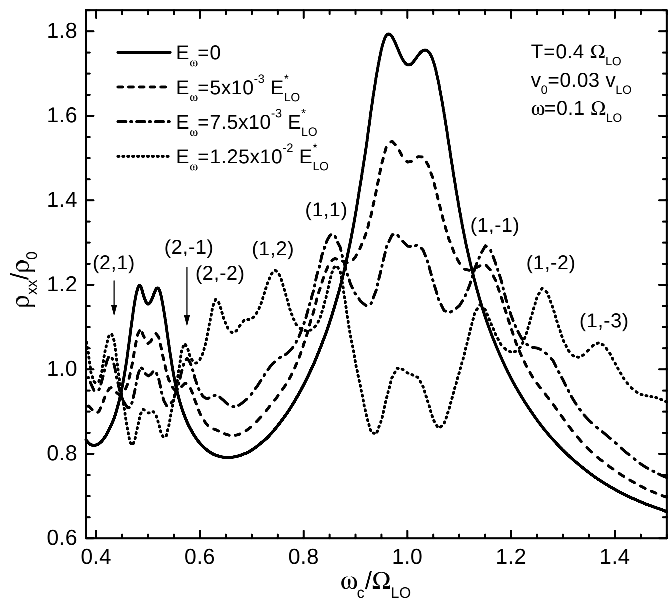

Fig. 3. The multiphoton-magnetophonon resonance in the longitudinal resistivity ρ_{xx} of a polar semiconductor driven by a dc bias velocity $v_0 = 0.03v_{\rm LO}$ and by a radiation field of frequency $\omega = 0.1\Omega_{\rm LO}$ having three different amplitudes $E_\omega = 0.005, 0.0075$, and $0.0125E^*_{\rm LO}$ in the T$_\perp$ configuration. The lattice temperature is $T = 0.4\Omega_{\rm LO}$. The polar optic phonon Fröhlich scattering is assumed to dominate. Here ρ_0 represents the resistivity of the system at the same lattice temperature with the same dc bias velocity in the absence of the magnetic field ($B = 0$) and in the absence of the radiation field ($E_\omega = 0$). The multiphoton peaks are indicated by two integers (M, n) with reference to the resonant condition $M\omega_{\rm c} + n\omega = \Omega_{\rm LO}$.

The calculated longitudinal resistivity $\rho_{xx}(v_0)$ as a function of $\omega_{\rm c}/\Omega_{\rm LO}$ in the case having dc bias $v_0 = 0.03v_{\rm LO}$ and ac-frequency $\omega = 0.1\Omega_{\rm LO}$, is plotted in Figure 3, for 3 different values of the ac field strengths $E_\omega = 0.005E^*_{\rm LO}, 0.0075E^*_{\rm LO}$, and $0.0125E^*_{\rm LO}$. Here $v_{\rm LO} \equiv (\Omega_{\rm LO}/m)^{1/2}$ and $E^*_{\rm LO} = (m\Omega^3_{\rm LO})^{1/2}/|e|$. All the features described in the case of nonpolar optic phonon scattering remain unchanged, and, as far as the oscillatory part is concerned, the relative hump heights and valley depths are essentially the same, except that in the case of nonpolar phonon scattering the oscillatory resistivity is superposed on a background part, which increases with increasing the magnetic field.

6 Conclusion

We have developed a balance-equation approach to terahertz-driven magnetotransport in semiconductors with a dc or slowly-varying electric field, an intense polarized radiation field of THz frequency and a uniform magnetic field, being in arbitrary directions and having arbitrary strengths, applied simultaneously in the system. These equations, which include all orders of multiphoton processes, have been applied to the examination of the effect of a terahertz radiation on the magnetophonon resonance of the longitudinal resistivity in the transverse configuration in nonpolar and polar semiconductors subjected to a dc bias.

We find that the previous magneto-phonon resonance peaks in the absence of the high-frequency field are suppressed by the irradiation of the terahertz field, while many new peaks, which correspond to multiple photon emission and absorption processes, emerge and may become quite distinct, at moderately strong radiation field. Although these multiphoton-magnetophonon resonance peaks may be somewhat smeared by further broadening of the Landau levels due to other mechanisms, it should be observable in a magnetically quantized semiconductor when it is exposed to an intense terahertz radiation.

This work was supported by the National Natural Science Foundation of China, the National and Shanghai Municipal Commissions of Science and Technology of China, and the Shanghai Foundation for Research and Development of Applied Materials.

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